Problem Solving: Two-Variable Relationships

LAUNCH (9 MIN)

Before
• Why would two friends choose different methods to present their information?

During
• What do the variables represent in Method 2?

After
• Is there a faster way to extend the table in Method 1?

PART 1 (8 MIN)

During the Intro
• Why would two banks have different options for accounts?
• Which deal seems better according to your first impression? Explain.

Before solving the problem
• Why would you want to find a pattern that describes the deal for Bank of Lotsabucks?
• Which model is most useful to show this relationship? Explain.

After completing the animated solution
• What is the constant in your equation? What does it represent?

PART 2 (7 MIN)

During the Intro
• Do you have to rewrite repeated multiplication with an exponent?

Before solving the problem
• How is this problem different from the example in Part 1?

After solving the problem
• How do your account totals help you predict which savings account is a better deal?

Xiao Says (Screen 3) Use the Xiao Says button to show students that exponents can increase a value very quickly.

PART 3 (8 MIN)

During the Intro
• How can both banks claim they are better than the other?

Before solving the problem
• Why are both equations written in terms of the dependent variable?

While solving the problem
• How can you find out how much money you will withdraw after 4 years?

After solving the problem
• Which account turned out to be the best deal?

Xiao Says (Screen 2) Use the Xiao Says button to highlight the point in time when the better deal switches from Lotsabucks to Wahoo.
• What does it mean when the graphs of two equations cross?

CLOSE AND CHECK (8 MIN)

• How can you compare two-variable equations?
• How can you write a comparison between the values of the variable you are comparing?
Problem Solving: Two-Variable Relationships

**LESSON OBJECTIVES**

1. Use variables to represent two quantities in a real-world problem that change in relationship to one another.
2. Evaluate expressions at specific values of their variables, including expressions that arise from formulas used in real-world problems.

**FOCUS QUESTION**

What types of problems are best solved using tables?

**MATH BACKGROUND**

Students have previously learned to write and solve one-variable equations and inequalities. They then learned how to identify and model two-variable relationships and compare them to relationships involving one variable. Students can describe how one variable depends on another variable using words, equations, graphs, and tables.

This lesson focuses on two types of relationships that students are able to write: linear and exponential equations. Although students are not familiar with the name for either type of equation, they have worked with linear and exponential expressions in the topic *Variables and Expressions*. Students will write equations that describe each relationship and use those equations to compare the two situations for various values of the independent variable. By comparing the behavior of two equations/tables/graphs, students can better write equations involving two variables in later lessons. Students will next use two-variable relationships when they write proportions and later solve systems of equations.

**LAUNCH (9 MIN)**

Objective: Describe how a table and equation can relate to the same problem.

**Author Intent**

Students analyze two different representations of a two-variable relationship. They analyze the methods of using a table and an equation to solve the same problem.

**Questions for Understanding**

**Before**
- Why would two friends choose different methods to present their information? [Sample answer: Each friend may prefer a different model to show information.]

**During**
- What do the variables represent in Method 2? [The variable \( l \) is the independent variable, the number of lawns mowed. The variable \( T \) represents the total savings, the dependent variable.]

**After**
- Is there a faster way to extend the table in Method 1? [Sample answer: You can find the difference for every 3 lawns and use that information to only make a row for every 3 lawns mowed.]

**Solution Notes**

Some students will choose Method 1 because they like seeing the total savings after each lawn is mowed. Ask students whether that is the most efficient way to solve this problem. Other students will choose Method 2 because you can solve for \( l \) when \( T \) is 300 or use systematic guess and check to find a large enough value for \( l \).
You can also show students how to rewrite Method 2 as an inequality. The two friends need at least $300 for the trip, so you can solve for all values of $l$ that result in a savings of at least 300.

**Connect Your Learning**

Move to the Connect Your Learning screen. In the Launch, students compared two methods to describe the same pattern. Discuss how an equation may be a better method in this situation. Use the Focus Question to remind students that they have primarily used tables in this topic to find the relationship between two variables, especially as a first step to writing an equation.

**PART 1 (8 MIN)**

Objective: Use variables to represent two quantities in a real-world problem that change in a linear relationship to one another and write an equation.

**Author Intent**

Students are presented with two options for savings accounts. They predict which account is a better deal and then write an equation that models the first option. This problem allows students to use whichever method they prefer to write an equation. The first option is an example of a linear equation, the type of relationship students have seen most often in this topic.

**Instructional Design**

Show the animation in the Intro to set up the three parts of this lesson. Let students share preconceptions and let the class vote on which savings account they think is a better deal.

Move to Screen 2 to begin working with one of the accounts. Use the blank Know-Need-Plan organizer to help students start to find a pattern and decide how best to write an equation that models the situation. You can call on students to come to the whiteboard and fill in each part.

**Questions for Understanding**

**During the Intro**

- Why would two banks have different options for accounts? [Sample answer: Each bank is presenting a deal that seems better in order to get more customers.]
- Which deal seems better according to your first impression? Explain. [Sample answer: Bank of Lotsabucks may have a better deal because they are paying a large amount every year.]

**Before solving the problem**

- Why would you want to find a pattern that describes the deal for Bank of Lotsabucks? [Sample answer: You can quickly find out how much money you will have after a certain number of years.]
- Which model is most useful to show this relationship? Explain. [Sample answers: An equation is most useful because you can plug in a value for the number of years to find out the total savings. A table is most useful because you can organize the total savings and compare it to the other account.]

**After completing the animated solution**

- What is the constant in your equation? What does it represent? [1; the amount you used to open the bank account]
Solution Notes

This problem has an animated solution that completes the Know-Need-Plan organizer and then steps students efficiently through making a table, describing the relationship, and writing an equation.

You may also want to open the Coordinate Grid from the Tools menu and let students graph the points in the table. Explain that a graph is useful to view the information in a different form but is not the most useful way to describe the pattern in this problem.

Students who are able to describe the relationship in words may need a Words to Equation organizer in order to write the equation.

Got It Notes

One way to present this lesson is to complete all three Examples and then have students repeat the comparison using all three Got Its.

Alternatively, you can compare this problem directly to the Example so that students can get used to seeing how equations are related to each other. They may not need to fully solve this problem if they realize they only need to replace 100 with 200.

If you show answer choices, consider the following possible student errors:

If students select A, they may be reversing the two quantities. Students who are leaving out the initial dollar may choose C. Show students who choose D that there is only one variable in this answer, which makes the total savings a constant.

PART 2

Objective: Use variables to represent two quantities in a real-world problem that change in an exponential relationship to one another and write an equation.

ELL Support

Beginning

Play the audio on Screen 2 of the Example. To assist students in completing the Know-Need-Plan graphic organizer, write the following sentence frames on the board:

I start with ____[$1]. At the end of each year, my money is ____ [doubled]. I need to write an ____ [equation] that relates the ____ [year] and the amount of money in the account. Help students complete the sentence frames to use in the Know and Need boxes. Then have students suggest ideas for the Plan box. Discuss the advantages of using a table and have them complete the sentence frame: Make a ____ [table] to show how much money is in the account for the first few years.

Intermediate

Play the audio on Screen 2 of the Example. To help students complete the Know–Need–Plan graphic organizer, ask questions to help them locate the information in the problem, such as: How much money do you start with? [$1] How often does your money double? [every year] What two things need to be related in the equation you write? [the year and the amount of money in the account] How would making a table be helpful? [I can see the amount of money present each of the years.]

Advanced

Have Advanced High learners work in pairs to discuss what information should be placed in the Know-Need-Plan graphic organizer. Have students discuss why each piece of information goes into the box it does and the importance of organizing information to solve a word problem.
**Author Intent**

In Part 1, students were introduced to two different deals for savings accounts and wrote an equation that modeled one of them. Now, they repeat the process for the other deal. This equation is exponential, which students may not have seen since the lesson *Expressions with Exponents.*

**Instructional Design**

Use the Intro to review exponents and notation as a class, which will hint that you can use powers to solve the Example.

On Screen 2, students can come to the whiteboard to fill out the blank Know-Need-Plan organizer. You can show them how the steps in the Plan box are identical to those in the previous example.

**Questions for Understanding**

**During the Intro**
- Do you have to rewrite repeated multiplication with an exponent? [Sample answer: No; writing powers may make the expression shorter or easier to work with.]

**Before solving the problem**
- How is this problem different from the example in Part 1? [Sample answer: In Part 1, you add the same amount each year. In this problem, you multiply by the same number each year.]

**After solving the problem**
- How do your account totals help you predict which savings account is a better deal? [Sample answer: The totals for Bank of Lotsabucks are much larger than those for Wahoo Savings and Loan. It appears that Lotsabucks is a better deal.]

**Xiao Says (Screen 3)** Use the Xiao Says button to show students that exponents can increase a value very quickly.

**Solution Notes**

Like the previous example, this problem has an animated solution that demonstrates how to organize the problem and then carefully and efficiently takes students through the steps of the solution. You can compare your class solution to this animation or use the animation once the class forms a plan to solve the problem.

Discuss the similarities and differences between the solution to this problem and that of Part 1. Pay particular attention to the difference in the two equations and the exponential nature of the equation that models Wahoo Savings and Loan.

**Differentiated Instruction**

**For struggling students**: Students found that the starting amount of $1 was an important part of the equation (the constant) in Part 1. Ask students to explain where the starting amount appears in the equation of this problem. They may mention that the starting amount does not appear in this equation because it was $1, and you do not need to multiply the equation by 1 according to the Identity Property of Multiplication. Have students rewrite the equation to specifically include the starting amount.

**For advanced students**: Have students plot points for this equation on a graph with the x-axis labeled from 1 to 10 and the y-axis from 1 to 1,100, with tick marks every 100 units. Have students connect the points in a smooth curve. Then use a different color and point plots from the previous Example, also connecting the points. After solving the Example for Part 3, students can share their graphs with the class. You may want to introduce the terms *linear* and *exponential* to describe the shapes of the graphs, though students will not work with functions until Grade 8.
Got It Notes

Follow the same pattern that you did in Part 1. You can either choose to compare this problem to the Example to help students compare the equations, or you can do all three examples and then come back to repeat the process for the Got Its.

If you show answer choices, consider the following possible student errors:

Students who choose A may not see a difference between tripling a number and adding three to it. If students choose B, they may not understand the meaning of the variable and think that they are multiplying the account total by 3. Students who select D are adding the starting amount to the correct total instead of multiplying it.

PART 3 (8 MIN)

Objective: Evaluate two expressions at the same value(s) of their variables and make a decision based on the results.

Author Intent

Students compare the two equations from earlier in the lesson. They use several values of the independent variable to make decisions about which bank account is better.

Instructional Design

Use the Intro to set up the final comparison of the two savings accounts. Move to Screen 2. You can call on students to substitute the first value into each equation and solve for the other variable. Have the class decide which account is better for that time period, and let a student drag the tile to the correct box. Repeat for the remaining tiles and then press the Check button. Any incorrect answers will snap back to the tile bank and present an opportunity to review solving equations.

Questions for Understanding

During the Intro

• How can both banks claim they are better than the other? [Sample answers: Both banks want customers; they may claim they are the best without actually being the best. The two deals are different, so one may be the best for some situations but not others.]

Before solving the problem

• Why are both equations written in terms of the dependent variable? [Sample answer: You are much more likely to know the independent variable, so you can solve for the dependent variable more easily this way.]

While solving the problem

• How can you find out how much money you will withdraw after 4 years? [Sample answer: You can plug in 4 for y in each equation and solve for T.]

After solving the problem

• Which account turned out to be the best deal? [Sample answer: Both accounts were a better deal for some of the tiles. It depends on the number of years you save your money.]

Xiao Says (Screen 2) Use the Xiao Says button to highlight the point in time when the better deal switches from Lotsabucks to Wahoo.

• What does it mean when the graphs of two equations cross? [Sample answer: That is the point where the two equations are the same, the two account totals are equal.]
Solution Notes

This problem also has an animated solution that calculates the total amounts for each savings account for each of the four parts of this problem. You can show the calculations instead of doing them as a class, or you can skip the calculations and show students the conclusion of the problem.

Error Prevention

Students may have already assumed from Parts 1 and 2 that Bank of Lotsabucks has the better deal because they only observed the account totals for the first few years. Make sure students properly substitute values into the equations they wrote earlier in the lesson.

Got It Notes

Be sure to remind students that the two equations presented model the real-world situations in the Got Its from Parts 1 and 2. Students should understand that this problem mirrors the Example but only focuses on one value of y.

You can use the solution of this problem to start a discussion about how rapidly an equation involving exponents grows. This prepares students to work with the graphs of exponential functions and sequences involving powers in high school.

CLOSE AND CHECK  (8 MIN)

Focus Question Sample Answer

When a problem has two related and unknown quantities, you can use a table to organize data. Assign a quantity to each column of the table, and you may see a pattern develop as you add data.

Focus Question Notes

Students may mention that any problem for which an equation is helpful is also a problem that uses tables. They might discuss how difficult it can be to write an equation from a word problem without first making a table to identify the pattern.

Make sure students understand that the independent variable should be in the first column of the table, and the dependent variable in the second column. While students may still be thinking about assigning variables that make the most sense in the context of the problem, some students may be ready to assign x and y for every situation.

Essential Question Connection

Students are now not only writing equations that use a second variable but also are solving for values of these variables. The Essential Question asks: “How are two-variable relationships different from one-variable relationships?” Use the questions below to remind students that they can solve a one-variable relationship for that variable, but you cannot solve a two-variable relationship unless you first substitute a value for one of the variables.

- How can you compare two-variable equations? [Sample answer: If the two-variable equations contain the same variables, you can substitute the same value for the independent variable and compare the values of the dependent variables.]

- How can you write a comparison between the values of the variable you are comparing? [Sample answers: You can find out how much larger one value is than another. You can find out how many times larger one value is than the other. You can write a ratio of the two values.]