Constant of Proportionality

LAUNCH (6 MIN)

Before
- How can you use the titles on the axes to help you understand the meaning of the graph?
- Does this graph show a proportional relationship? How can you tell?

During
- What do the three points, A, B, and C represent in the context of this problem?
- Can you figure out an employee’s hourly rate using the graph? Which point should you use?

After
- Would the point (15, 1500) lie on the graph? How do you know?

KEY CONCEPT (3 MIN)

The constant of proportionality can also be called the unit rate, and it is found by simplifying any ratio of $y$ to $x$ in a proportional relationship.

Although there are two reciprocal rates possible in this situation, only the $y : x$ ratio gives the constant of proportionality.

PART 1 (6 MIN)

Dana Says (Screen 1) Use the Dana Says button to emphasize that this new idea of a constant of proportionality is really just a unit rate (when the quantities have different units).

- When does a stack of books have a unit weight? Where would you see a stack like this?

After finding the constant of proportionality
- How does finding the constant of proportionality help you find the weight of 11 books?

PART 2 (6 MIN)

Before solving the problem
- What is the meaning of the constant of proportionality in this problem?
- How will you determine if the constants of proportionality are the same in both recipes?

After solving the problem
- Why would two cookie recipes have different constants of proportionality?

PART 3 (6 MIN)

Dana Says (Screen 1) Use the Dana Says button to help students recognize that if two quantities do not have a proportional relationship, there is no constant of proportionality.

After solving part (a)
- What does the constant of proportionality in this problem mean?
- How can you estimate the amount of money raised for 600 tickets? For 2,500 tickets?

PART 4 (6 MIN)

Before solving the problem
- How can you tell that this graph has a constant of proportionality?

After solving the problem
- How can you use the constant of proportionality to figure out the number of wing beats a hummingbird makes in any number of seconds?

CLOSE AND CHECK (7 MIN)

- What does the existence of a constant of proportionality tell you?
- In a situation involving a proportional relationship between the cost in dollars and the amount purchased, what does the constant of proportionality tell you?
LESSON OBJECTIVES

1. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships.

2. Explain what a point \((x, y)\) on the graph of a proportional relationship means in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) where \(r\) is the unit rate.

FOCUS QUESTION

What is a constant of proportionality? What does the constant of proportionality tell you?

MATH BACKGROUND

In the previous topic, students worked with unit rates and found that equivalent ratios have the same unit rate. Now that students can recognize proportional relationships from both tables and graphs, in this lesson they will begin to describe proportional relationships. A proportional relationship exists between two ratios when they are equivalent to each other. Students learn the definition of a constant of proportionality and what it means in the context of a problem. A constant of proportionality is the factor or constant value in a proportional relationship. Students recognize that if two ratios are equivalent, then they have the same constant of proportionality, so the constant of proportionality is the unit rate. They learn how to find the constant from both a table and graph. In the next lesson, students will describe proportional relationships using equations that are based on the constant of proportionality.

LAUNCH (6 MIN)

Objective: Explain the importance of the unit rate on the graph of a proportional relationship.

Author Intent

In the last lesson, students learned that a line passing through the origin shows a proportional relationship. In this Launch, which shows the graph of a proportional relationship, students interpret the meaning of the three points that are graphed. One of the points shows the unit rate.

Instructional Design

You may need to explain to students that some people are paid an annual salary while others are paid on an hourly basis.

Questions for Understanding

Before

- How can you use the titles on the \(x\)- and \(y\)-axes to help you understand the meaning of the graph? [The titles indicate that the graph shows the amounts of money earned for working different numbers of hours.]

- Does this graph show a proportional relationship? How can you tell? [Sample answer: Yes; it shows a line going through the origin.]

During

- What do the three points, A, B, and C represent in the context of this problem? [Point A shows that for 1 hour, an employee earns $10. Point B shows that for 5 hours, an employee earns $50. Point C shows that for 8 hours, an employee earns $80.]
Can you figure out an employee’s hourly rate using the graph? Which point should you use? [Sample answer: Yes; I can tell that an employee earns $10 for each hour worked. Point A is easiest to use because it already expresses a unit rate.]

After

Would the point (15, 1500) lie on the graph? How do you know? [No; for every point on the line, the y-value is 10 times the x-value.]

Solution Notes

Students may choose any of the three points as long as they justify why it is most useful. For employees who get paid by the hour, Point A is probably most useful. For employees who work 5 or 8 hours per day, Points B and C may be most useful, respectively.

Connect Your Learning

Move to the Connect Your Learning screen. Start a discussion about the graph in the Launch; make sure students see why it shows a proportional relationship. Ask them what kinds of information they can tell from the graph. Listen for a student to mention that a new manager makes $10 per hour. Some students may recall that this is a rate, because it compares two quantities with different units.

KEY CONCEPT

Beginning

Have students look at and listen to the introductory paragraph of the Key Concept. Ask them to write down some key words in the paragraph. Write simple definitions on the board and have students write each definition next to the correct word. Spanish-speaking students could also write the Spanish equivalents. [quantity (cantidad): amount; proportion (proporción): relationship between two quantities; constant (constante): always the same; multiple (múltiplo): the result of multiplying by a certain number; unit (unidad): one of something]

Intermediate

Have students watch and listen to the animation in the Key Concept. Then write the following sentences on the board and have students take turns writing words or numbers in the blanks to make the sentences true. There is/are ___ brick(s) with length ___ centimeters. The proportion total length to number of bricks is ___ to ___, so the constant of ___, is ___. The ___ of proportionality is ___ centimeters per ___.

Advanced

Go over the definitions and the animation in the Key Concept. Let students try some of the Examples and Got Its in Parts 1–4. Then ask students to write their own problems involving proportional relationships using the problems in the lesson as models. Make sure they are correctly incorporating the constant of proportionality in their problems. Let them trade problems with a partner and solve.

Teaching Tips for the Key Concept

The table in this Key Concept is very similar to the tables students made in the lesson Proportional Relationships and Tables, and the ratio that students found is now given a formal name: constant of proportionality.

The constant of proportionality can also be called the unit rate, and it is found by simplifying any ratio of y to x in a proportional relationship. Play the animation to strengthen students’ understanding that the constant of proportionality is the unit rate and to make the connection between the situation and the data in the table.
Although there are two reciprocal rates possible in this situation, only the $y : x$ ratio gives the constant of proportionality.

**PART 1 (6 MIN)**

Objective: Identify the constant of proportionality (unit rate) in diagrams of proportional relationships.

**Author Intent**

Students are told that two real-world quantities have a proportional relationship and are asked to find the constant of proportionality. Students can find any of the ratios given. Once they find the constant of proportionality, they use it to find the $y$-value that corresponds to a given $x$-value.

**Questions for Understanding**

- **Dana Says (Screen 1)** *Use the Dana Says button to emphasize that this new idea of a constant of proportionality is really just a unit rate (when the quantities have different units). See if students remember that unit rates are displayed beneath each item in most grocery stores, and help you compare brands.*

  - When does a stack of books have a unit weight? Where would you see a stack of books like this? *[Sample answer: The books may have the same unit weight if they are all the same book. This situation is common for people working in a bookstore.]*

After finding the constant of proportionality

  - How does finding the constant of proportionality help you find the weight of 11 books? *[The constant of proportionality represents the weight of 1 book. I can multiply the weight of 1 book by 11 to find the weight of 11 books.]*

**Error Prevention**

If students want to find the unit rate for books per pound, show them that this unit rate is not as helpful to find the weight of 11 books. Note that this unit rate is actually very useful if you wanted to know the number books in a stack of a given weight.

**Got It Notes**

If you show answer choices, consider the following possible student errors:

- Students may select A if they see the 3 piles of shoeboxes. They may choose C if they divide the height by the number of piles or find the total number of shoeboxes.
- Students may choose D if they multiply the number of inches and number of shoeboxes.

**PART 2 (6 MIN)**

Objective: Identify and/or interpret the constant of proportionality (unit rate) in verbal descriptions of proportional relationships.

**Author Intent**

Students examine two ratios to determine whether they have the same constant of proportionality. Students will realize that ratios with the same constant of proportionality are equivalent ratios.

Note: Students need to know that there are 12 items in 1 dozen.
Questions for Understanding

Before solving the problem

• What is the meaning of the constant of proportionality in this problem? [It is the unit rate, number of cookies per cup of flour.]

• How will you determine if the constants of proportionality are the same in both recipes? [Sample answer: I will write a ratio of number of cookies to number of cups of flour for both recipes and simplify them. If they are equivalent, then I know the constants of proportionality are the same.]

After solving the problem

• Why would two cookie recipes have different constants of proportionality? [Sample answer: Some types of cookies use more flour than others.]

Differentiated Instruction

For struggling students: Use manipulatives (color tiles, beans) to show the ratios in both cookie recipes. For example, if you can make 36 cookies with 2 cups of flour, ask how many cookies you can make with 1 cup of flour. Break up the 36 color tiles into 2 equal piles to find that 1 cup of flour makes 18 cookies.

For advanced students: Have students see if there exists a constant of proportionality between their heights and their arm lengths (for example). Ask students what it means for the constants of proportionality to be a little different or very different. Talk about the possibility of error margins when measuring.

Got It Notes

Students may need help figuring out the order of the terms in the rate. Stress that the point is not to choose which rate is more useful in the situation but to decide whether the ratios are equivalent. You can show both miles per minute and minutes per mile and emphasize that in each case the rates must be consistent with units.

PART 3 (6 MIN)

Objective: Identify the constant of proportionality (unit rate) in tables of proportional relationships.

Author Intent

Students examine the values in a table to determine if the quantities have a proportional relationship and to find the constant of proportionality. They then use the constant to find another equivalent rate and solve a problem.

Questions for Understanding

Dana Says (Screen 1) Use the Dana Says button to coach students until they recognize that if two quantities do not have a proportional relationship, there is no constant of proportionality to find.

After solving part (a)

• What does the constant of proportionality in this problem mean? [It is the cost per ticket.]

• How can you estimate the amount of money raised for 600 tickets? For 2,500 tickets? [Sample answer: The amount of money raised for 600 tickets will be between the amounts for 500 and 750 tickets. The amount of money raised for 2,500 tickets will be much more than the values in the table.]

Got It Notes

If you show answer choices, consider the following possible student errors:
Students might select B if they divide decimals incorrectly or misplace the decimal point. They may choose C if they find the ratio of number of gallons to number of seconds. They may choose D if they calculate one of the ratios incorrectly.

**PART 4 (6 MIN)**

Objective: Identify the constant of proportionality (unit rate) in graphs of proportional relationships.

**Author Intent**

For the first time, students find the constant of proportionality directly from a graph rather than from a table of values. Once they find the constant of proportionality, they interpret its meaning in the context of the real-world situation. Since students do not yet know how to use slope, they need to find the ratio \( \frac{y}{x} \) for any point on the line.

**Questions for Understanding**

**Before solving the problem**

- How can you tell that this graph has a constant of proportionality? [It is a straight line through the origin, so it is a proportional relationship.]

**After solving the problem**

- How can you use the constant of proportionality to figure out the number of wing beats a hummingbird makes in any number of seconds? [Sample answer: I can multiply the constant of proportionality, 80, by any number of seconds to find the number of wing beats.]

**Solution Notes**

Note that a graph is a more efficient way to identify a proportional relationship than is a table. While students do find the constant of proportionality while calculating ratios for each row of a table, they only need to find one ratio from the graph.

**Differentiated Instruction**

*For struggling students*: Support students in reading the titles on the x- and y-axes to comprehend the graph. Students will use the titles on the axes to help them figure out what the point (1, 80) means: “In 1 second, there are 80 beats.”

*For advanced students*: Challenge students to write an equation that represents this situation. Students should write \( y = 80x \). This prepares students to describe proportional relationships using equations, which they will learn in the next lesson.

**Got It Notes**

If you show answer choices, consider the following possible student errors:

Students may choose A if they find the ratio of time to distance. They may select D if they do not understand how to identify a proportional relationship from a graph.

**CLOSE AND CHECK (7 MIN)**

**Focus Question Sample Answer**

Sample: The constant of proportionality describes the relationship between two quantities that have a proportional relationship. It is the ratio of \( y \) to \( x \), or a unit rate. It tells you the constant multiple between the two quantities.

**Focus Question Notes**

Rich student answers should identify the different ways they found a constant of proportionality in this lesson: by looking for equivalent ratios in a problem situation,
comparing the ratios between the $x$- and $y$-columns of a table, and in a graph. Students may also describe the recipe example in which the two constants of proportionality are different, so the recipes are not for different batch sizes of the same cookie.

**Essential Question Connection**

This lesson addresses the Essential Question about distinguishing proportional relationships from disproportional relationships by introducing the existence of a constant of proportionality in proportional relationships.

- What does the existence of a constant of proportionality tell you? [Sample answer: The existence of a constant of proportionality tells you that the two quantities have a proportional relationship.]

- In a situation involving a proportional relationship between the cost in dollars and the amount purchased, what does the constant of proportionality tell you? [Sample answer: In such a situation, the constant of proportionality tells you the cost for one unit of the item. This is known as the unit rate, or in this situation, the unit cost.]

- Now that you’ve looked at the constant of proportionality and how it relates to tables and graphs, what do you think the next lesson will address? [Sample answer: Equations that have a constant of proportionality.]