Proportional Relationships and Equations

LAUNCH (6 MIN)

Before
- How is the number 40 used in relation to \( x \) in each equation?

During
- If you saw the graphs of these two equations, how would you know if either of them represents a proportional relationship?

After
- How can you use the equation \( y = 40x \) to figure out the amount of money you earn in a 40-hour week? What does each variable represent?

KEY CONCEPT (3 MIN)

You can make a table using the equation from the Launch to help students not only understand the general equation \( y = mx \) but also see how the constant of proportionality affects the equation.

- How would the constant of proportionality change if the total number of hours worked each week were 50 instead of 40? How does that affect the equation?

PART 1 (6 MIN)

Dana Says (Screen 1) Use the Dana Says button to emphasize that the equation \( y = 8.5x \) does have a constant of proportionality; it is 8.5.

After solving part (b)
- What would the equation \( y = 8.5x \) look like if it were graphed? How do you know?
- How is the constant of proportionality a unit rate?

PART 2 (6 MIN)

Before sorting the equations
- Does each situation give the unit rate? Explain.

While solving the problem
- Do you expect the constant of proportionality to be greater than the numbers in the situation or less than the numbers in the situation?

PART 3 (6 MIN)

Before writing the equation
- Why would you want to know the exchange rate for U.S. dollars to Euros?
- What value do you need to find from the table in order to write the equation?

Dana Says (Screen 2) Use the Dana Says button to point out that being able to write an equation from a table of values is very helpful. Start a discussion about the benefits of tables versus equations.

PART 4 (6 MIN)

Before solving the problem
- How can you estimate the number of goalies?

While solving the problem
- How can you use the ratio \( \frac{2}{30} \) to find the number of goalies needed for 915 players?

CLOSE AND CHECK (7 MIN)

- How can you tell if an equation represents a proportional relationship?
- Which method of distinguishing a proportional relationship do you prefer?
Proportional Relationships and Equations

LESSON OBJECTIVES

1. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships.
2. Represent proportional relationships by equations.

FOCUS QUESTION

How can you tell if an equation shows a proportional relationship between two quantities? How can you identify the constant of proportionality in an equation that represents a proportional relationship?

MATH BACKGROUND

The third model students will use in this topic to represent a proportional relationship between two quantities is an equation. They use what they learned about the constant of proportionality to write an equation in the general form \( y = mx \), where \( m \) represents the constant of proportionality. They learn to identify equations that represent proportional relationships and then generate equations that show proportional relationships by finding the constant of proportionality from data. In future courses, students will use the term slope to describe the constant of proportionality in the context of linear equations of the form \( y = mx \).

This lesson also defines proportion and purposely avoids the method of "cross multiplication" in order to further students' understanding of proportional relationships and bolster their ratio reasoning skills.

LAUNCH (6 MIN) ____________________________________________________________

Objective: Describe the usefulness of proportions for real-world situations.

Author Intent

Students are shown two equations, only one of which shows a proportional relationship. They decide which equation would be most useful to an employee. Students will understand that employees would be interested in knowing the hourly rate they get paid. As students examine the two different types of equations, they recognize which equation shows a proportional relationship.

Instructional Design

Students need to be aware that this problem assumes a typical workweek consists of 40 hours.

Questions for Understanding

Before

- Both equations have the number 40 and the variable \( x \). How is the number 40 used in relation to \( x \) in each equation? [In the first equation, 40 is added to \( x \). In the second equation, 40 is multiplied by \( x \).]

During

- If you saw the graphs of these two equations, how would you know if either of them represents a proportional relationship? [You can see whether either equation is a straight line through the origin.]

After

- How can you use the equation \( y = 40x \) to figure out the amount of money you earn in a 40-hour week? What does each variable represent? [The \( x \)-value represents the hourly rate you earn. The \( y \)-value represents the total amount of
money earned. You can substitute your hourly rate for \( x \) to find the money you earn in a week.]

**Connect Your Learning**

Move to the Connect Your Learning screen. Start a discussion about which equation in the Launch shows a proportional relationship. Encourage students to make a table of values for each equation, or graph each equation, to identify which shows a proportional relationship. Some students may recognize 40 as the constant of proportionality.

**KEY CONCEPT** (3 MIN)

**Teaching Tips for the Key Concept**

Students sometimes find it easier to understand a general formula if they see the relationship in the form of a table. You can make a table using the equation from the Launch, \( y = 40x \), to help students not only understand the general equation \( y = mx \) but also see how the constant of proportionality affects the equation.

Students should understand the difference between dependent and independent variables in an equation. The value of the dependent variable \( y \) is a result of the value chosen for the independent variable \( x \).

**Questions for Understanding**

- How would the constant of proportionality change if the total number of hours worked each week were 50 instead of 40? How does that affect the equation? [The constant of proportionality would change from 40 to 50. The new equation would be \( y = 50x \).]

**PART 1** (6 MIN)

Objective: Identify and explain the constant of proportionality (unit rate) in equations of proportional relationships and evaluate equation.

**Author Intent**

Students find the constant of proportionality from an equation of the form \( y = mx \). They explain the meaning of the constant of proportionality in the context of the situation and use it to make decisions and find unknown values.

**Questions for Understanding**

**Dana Says (Screen 1)** Use the Dana Says button to emphasize that the equation \( y = 8.5x \) does have a constant of proportionality; it is 8.5. It is the number that each \( x \)-value must be multiplied by in order for the product to be the corresponding \( y \)-value. Emphasize the use of the word constant in the term.

After solving part (b)

- What would the equation \( y = 8.5x \) look like if it were graphed? How do you know? [Sample answer: It would be a straight line through the origin. Every proportional relationship can be modeled with a straight line through the origin.]
- How is the constant of proportionality a unit rate? [Sample answer: It is the ratio of the cost to the number of tickets where the denominator is 1. Therefore, it is the cost per unit (ticket).]

**Solution Notes**

This problem does not show a table of values or a graph with points on it. Some students may benefit from making a table of values and/or a graph to fully comprehend the problem.
Differentiated Instruction

For struggling students: You can use index cards to represent movie tickets. On each card, write $8.50. Give students different numbers of cards to represent different numbers of movie tickets. They can find the number of tickets, the total price of the tickets, and the cost of one ticket. They should see that the number of tickets and the total price may change, but what always stays the same is the price of each ticket, which is the constant of proportionality.

For advanced students: Have students rewrite the problem statement without using the equation $y = 8.5x$, which involves a greater understanding of the situation. Students should realize that 8.5, or $8.50, represents the cost of a movie ticket, and that you don’t know how many movie tickets have been sold.

Got It Notes

This Got It is similar to the Launch. In the Launch, the constant of proportionality is 40 because that was the number of hours in a workweek. Here, the constant of proportionality is 4 because there are 4 sides to every square. If you know the length of one side of a square, you can use the formula $P = 4s$ to find the perimeter.

If you show answer choices, consider the following possible student errors:

Students may choose A or C if they plugged a side in for $P$ instead of $s$. They might select C or D if they found the ratio in the wrong order.

PART 2 (6 MIN)

Objective: Represent a proportional relationship given by a verbal description with an equation.

Author Intent

Students are given three real-world situations that can be represented by proportional equations and find the equation for each situation. This is the first time that students write an equation to model a proportional relationship. They find the constant of proportionality and substitute it into the equation $y = mx$.

Instructional Design

Have individual students read each problem. You can call on other students to drag the correct tile to each equation. Since there are more equations than situations, any equations that do not have a match should be dragged to the recycle bin.

Questions for Understanding

Before sorting the equations

- Does each situation give the unit rate? Explain. [No; only the first situation gives a unit rate. You can find the unit rate for the others.]

While solving the problem

- Do you expect the constant of proportionality to be greater than the numbers in the situation or less than the numbers in the situation? [Less than; you find the constant of proportionality by finding a ratio, which is division.]

Solution Notes

Play the animated solution to reinforce why finding the unit rate in each situation is critical for finding the correct equation. The solution steps through each situation and translates the unit rate into an equation.

Error Prevention

Students will sometimes try to solve a problem without gathering all the information they need. In this problem, students may try to write an equation using only the
numbers given. Encourage students to recognize that they need to find the constant of proportionality, which is the unit rate, in order to select the correct equation for each situation.

**Got It Notes**

If you show answer choices, consider the following possible student errors:

Students may choose A because they see the number 5 in the problem. Students may choose B because the problem uses $2.50. If they select C, they are finding the reciprocal rate.

**PART 3 (6 MIN)**

Objective: Represent a proportional relationship given in a table with an equation.

**ELL Support**

On the Student Companion page for the Part 3 Got It, there are two tasks for students to complete and discuss:

- What does currency exchange mean?
  - Have you ever traveled to a place where the currency was not the U.S. dollar?

**Beginning**

Have students discuss what currency exchange means. Are they bringing some knowledge of this from experience? If students are struggling with defining this term, analyze the two words that make up the term—currency and exchange. What does currency mean? What does exchange mean? Then have students discuss their answers, making changes based on their new understanding of currency exchange.

**Intermediate**

Have students discuss how they applied their background knowledge to answer this problem. Did they recognize currency? Did they have personal experience and knowledge of currency exchange?

**Advanced**

Have students revise their answers after sharing individual responses.

**Author Intent**

Students find the constant of proportionality from a table and use it to write an equation. Previously, students solved similar problems using conversion factors when they learned about unit rates and reciprocal rates.

**Instructional Design**

Use the Intro to help students understand how to exchange money from one currency to another and demonstrate how a conversion factor works as a constant of proportionality. Move to Screen 2 to have students write an equation from a different part of the table.

**Questions for Understanding**

**Before writing the equation**

- Why would you want to know the exchange rate for U.S. dollars to Euros?
  [Sample answer: If you are traveling to a country that uses Euros, you would want to know how many Euros your dollars are worth.]

- What value do you need to find from the table in order to write the equation?
  How will you find that value? [Sample answer: I need to determine the unit rate, which I can get by finding any ratio from the table.]
Dana Says (Screen 2) Use the Dana Says button to point out that being able to write an equation from a table of values is very helpful. Start a discussion about the benefits of tables versus equations. Tables tell you some facts very quickly, but make sure students understand that a table has limits. Tables have a limited number of rows, so they can only display a finite number of ordered pairs, whereas equations work for any value.

**Solution Notes**

Students may also write \( y = \frac{3}{4}x \) as an equation. Discuss whether the equation is more useful when expressed with a fraction or with a decimal, since money is expressed using decimals.

**Got It Notes**

This problem uses the same table of values from the Example. Now students switch the dependent and independent variables. They may recognize that it is just the reciprocal rate, but encourage students to use that fact as a check instead of a solution method. If you show answer choices, consider the following possible student errors:

Students may choose A if they find the same ratio as they found in the Example. If students choose B or C, they are finding the unit rate incorrectly.

**PART 4 (6 MIN)**

Objective: Represent a proportional relationship with a proportion and solve the proportion.

**Author Intent**

Students learn the definition of a proportion and how to set up and solve a proportion to answer a question involving quantities that have a proportional relationship. This problem purposely avoids cross multiplication to focus on ratio reasoning.

**Instructional Design**

Play the Intro so that students understand the definition of a proportion as well as how to set up and solve one. Stress that this is a new way to find an equivalent ratio when you know one term of the new ratio, and that it only takes one step to solve the equation using this method.

On Screen 2, have students come to the whiteboard and write a proportion that models the situation. Call on another student to solve the equation.

**Questions for Understanding**

**Before solving the problem**

- How can you estimate the number of goalies? [Sample answer: You know the number of goalies is much less than the total number of players. Since 2 is less than 10% of 30, there will be fewer than 90 goalies.]

**While solving the problem**

- How can you use the ratio \( \frac{2}{30} \) to find the number of goalies needed for 915 players? [Sample answer: I can set up and solve a proportion: \( \frac{2}{30} = \frac{x}{915} \).]

- How do you solve this proportion once you set it up? [Use multiplication to undo dividing by 915.]
**Solution Notes**
The provided solution has two methods: using a proportion and finding an equivalent ratio. Stress that Method 1 involves solving an equation, while Method 2 involves reasoning and may include an intermediate step of finding the unit rate. Let students choose the method they prefer.

**Error Prevention**
Remind students that they must be consistent with which quantity is the numerator and which is the denominator when setting up a proportion. In this case, the number of goalies is the numerator. They may want to write the ratio in words before setting up the proportion \( \frac{\text{number of goalies}}{\text{number of players}} \).

**Got It Notes**
Have students estimate their answers before solving as a test-taking strategy. If they realize that the answer is much less than 890, they can eliminate C and D.

If you show answer choices, consider the following possible student errors:
If students flip one of the ratios, they may choose D. Students who select C are probably using addition and/or subtraction to solve.

**Got It 2 Notes**
Help students reason that flipping both ratios keeps the equation true. If the ratios are equivalent, then their reciprocal rates are also equivalent.

**CLOSE AND CHECK (7 MIN)**

**Focus Question Sample Answer**
Sample: An equation shows a proportional relationship between two quantities if it can be written in the form \( y = mx \). The constant of proportionality is the coefficient of \( x \) in the equation \( y = mx \).

**Focus Question Notes**
Make sure students identify the coefficient of \( x \) in the equation \( y = mx \) as the unit rate in the problem situation. In this topic, the coefficient represented the cost of one movie ticket, calories per serving, a currency exchange rate, and several other unit rates.

**Essential Question Connection**
Studying an equation is a good way to determine if two quantities have a proportional relationship. This relates to the Essential Question about distinguishing proportional relationships from ones that are not proportional. Use the following questions to connect the Essential Question to equations. You may want to remind students that being able to take a problem situation and represent it with mathematical symbols is an important mathematical practice.

- How can you tell if an equation represents a proportional relationship? [Sample answer: If an equation is in the form \( y = mx \), then it represents a proportional relationship.]
- Which method of distinguishing a proportional relationship—using a table, graph, or equation—do you prefer? [Sample answer: The equation, because you can immediately tell if the equation is in the form \( y = mx \).]