Problem Solving: Proportional Relationships

LAUNCH (8 MIN)

**Before**
- What length will you choose the side of one square on your grid paper to represent?

**During**
- How many rectangular faces will your net have? Will any of the rectangles be identical?
- How do the net and the actual dimensions show a proportional relationship? Use the dimensions of the 9 by 12 wall in your explanation.

**After**
- How could you use your net to find the total area of the ceiling and walls of the addition?
- Would your net be a different shape if the given scale for the model was 1 in = 6 ft?

**PART 1** (8 MIN)

**Dana Says (Screen 1)** Use the Dana Says button to remind students that they know numerous ways to determine if a relationship is proportional. Remind students that it is good to be flexible, and comfortable with a variety of methods of problem solving.

**While solving the problem**
- What equation can you write to represent the situation about the dog bone? How can you tell that it does not represent a proportional relationship?
- What equation can you write to represent the situation about the party hat? How can you tell that it represents a proportional relationship?

**PART 2** (8 MIN)

**Dana Says (Screen 1)** Use the Dana Says button to explain to students that, in a blueprint, a break in the line representing the wall shows a doorway.
- Why is it common to carefully mark doorways in blueprints?

**Before solving the problem**
- What information do you need to solve the problem? How can you find that information?

**While solving the problem**
- The width of the section is 5 ft. Does that mean the bed won’t fit?

**After solving the problem**
- Both dimensions of the bed are longer than the width of the hallway or the door. How do you get the bed down the hallway and through the door to put it in the bedroom?

**PART 3** (8 MIN)

**Before solving the problem**
- How can you compare these exchange rates?

**Dana Says (Screen 1)** Use the Dana Says button to explain that exchange rates at different banks are often different. Even if the difference is small, travelers will still want the best exchange rate.
- Why would you want to know which bank has a better exchange rate?

**CLOSE AND CHECK** (8 MIN)

- Use one real-life example from this lesson to describe what constant of proportionality means.
- Suppose you are given a table of values. How could you determine if the table shows a proportional relationship without graphing the points?
- Suppose you are given an equation. How could you determine if the equation shows a proportional relationship?
Problem Solving: Proportional Relationships

LESSON OBJECTIVES

1. Solve problems involving scale drawings of geometric figures.
2. Recognize and represent proportional relationships between quantities.
3. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships and use to solve problems.

FOCUS QUESTION

In what ways can you represent proportional relationships? How can knowing how to represent proportional relationships in different ways be useful in solving problems?

MATH BACKGROUND

In this lesson, students draw on the knowledge that they have gained to represent proportional relationships using tables, graphs, equations, and scales. They use a scale to draw a net (the two-dimensional representation of a three-dimensional figure) of a room, which touches on their knowledge of surface area from Grade 6. Students distinguish situations that show proportional relationships from situations that do not. They solve a problem involving the scale drawing of a house to figure out whether an item of furniture will fit in a certain place. Finally, they solve problems involving exchange rates of different types of money, which they learned about in the lesson Proportional Relationships and Equations.

After completing this final lesson in this topic, students are prepared for future topics that require an understanding of proportional reasoning, most importantly the next topic, Percent. Proportional reasoning is a concept that students will be using in future math topics, especially in algebra, and is relevant in everyday situations.

LAUNCH (8 MIN)

Objective: Create a scale drawing given a scale of a model.

Author Intent

Students use the scale factor given and apply it to solving a problem. They recall their knowledge of nets to make a net that is a scale drawing of the room shown.

Instructional Design

Students may need you to review that a net is a two-dimensional sketch that represents a three-dimensional object. When students see a three-dimensional figure in a two-dimensional format, it helps them identify the various shapes that make up the three-dimensional figure. Nets are often used to find surface area.

You can open the Coordinate Grid from the Grids and Organizers menu on the Tools landing page to let students explore at the whiteboard while other students work individually or in small groups.

Questions for Understanding

Before

• What length will you choose the side of one square on your grid paper to represent? Explain. [Sample answer: 1 inch; the scale of the model given is 1 inch = 3 feet.]
**During**

- How many rectangular faces will your net have? Will any of the rectangles in your net be identical? Explain. [Sample answer: My net will have five rectangular faces. Two pairs of walls are identical.]

- How do the net and the actual dimensions show a proportional relationship? Use the dimensions of the 9 by 12 wall in your explanation. [Sample answer: The actual dimensions of the wall are 9 feet by 12 feet. The dimensions on the scale drawing are 3 inches by 4 inches. \( \frac{9}{12} \) and \( \frac{3}{4} \) are equivalent ratios and therefore have a proportional relationship.]

**After**

- How could you use your net to find the total area of the ceiling and walls of the professor’s addition? [Sample answer: I can label my net with the actual dimensions (9 ft or 12 ft) and then use them to find the sum of the areas of the five rectangles.]

- Would your net be a different shape if the given scale for the model was 1 in = 6 ft? Explain. [Sample answer: No; the net would look exactly the same. A change in the scale would only affect the actual size, not the shape of the net.]

**Connect Your Learning**

Move to the Connect Your Learning screen. Start a discussion about the various real-life situations in this topic in which proportional relationships have appeared. Listen for a student to describe the cookie recipe example as a situation in which a lack of a proportional relationship alerted the class that the recipes are not for the same cookie.

**PART 1 (8 MIN)**

Objective: Recognize and represent proportional relationships between quantities

**Author Intent**

Students determine whether different pairs of quantities have a proportional relationship or not. The situations are explained in terms of variables \( x \) and \( y \). Therefore, students need to translate from words to a model (table, graph, or equation) to identify which situations are proportional relationships.

**Instructional Design**

Have student volunteers come to the whiteboard to drag the relationships to the correct categories. Any incorrect tiles will snap back to the tile bank when you click the Check button.

**Questions for Understanding**

**Dana Says (Screen 1)** Use the Dana Says button to remind students that they know numerous ways to determine if a relationship is proportional. They can use a graph, table, or equation. You might challenge students to identify the representation with which they are least comfortable, and use it for at least one of the solutions. Remind students that it is good to be flexible, and comfortable with a variety of methods of problem solving.

**While solving the problem**

- What equation can you write to represent the situation about the dog bone? How can you tell that it does not represent a proportional relationship? [Sample answer: I can write the equation \( y = x + 3 \). It does not represent a proportional relationship because you cannot write it in the form \( y = mx \).]
What equation can you write to represent the situation about the party hat? How can you tell that it represents a proportional relationship? [Sample answer: I can write the equation \( y = 2x \). It represents a proportional relationship because I made a table of values, and the rows have equivalent ratios.]

**Solution Notes**

The provided solution shows multiple ways to examine the situations. Some situations use a table of values, while others use the Words to Equation organizer to write an equation that models the relationship. Students could also make a graph from a table and see whether it is a straight line through the origin. Point out that writing equations may be the most efficient way to solve this problem because you can sort the equations without substituting any values.

**Differentiated Instruction**

For struggling students: Students sometimes need support when translating from words to algebraic symbols or vice versa. Review vocabulary that applies to addition, subtraction, and multiplication, such as *addend*, *sum*, *minuend*, *subtrahend*, *difference*, *products*, and *factors*.

For advanced students: Challenge students to write equations for all of the situations in the problem (assuming they have not done so already). Then ask students to write new situations that could accompany each of the five equations. This will help students reflect on the difference between situations that involve proportional relationships and ones that do not.

**Got It Notes**

Once students remember that a scale on a map is a constant of proportionality, they should conclude that the relationship is proportional.

**PART 2 (8 MIN)**

Objective: Solve real-world problems involving scale drawings of geometric figures including computing actual lengths and areas from a blueprint.

**ELL Support**

**Beginning**

Read the Got It aloud and have students read along. Work through the solution as a group. Then pair Beginning learners with a native speaker and have partners work together to explain the solution steps. Encourage Beginning learners to point to the floor plan or use familiar words to help them explain the solution, as needed. Remind them to request assistance when they need it.

**Intermediate**

Read the Got It aloud and have students read along. Before working through the solution, have pairs work together to explain their solution ideas. Have students assist their partners in finding words when necessary, using synonyms, and speaking with greater detail.

**Advanced**

Have students read the Got It independently and consider a plan to solve the problem. Then have pairs of students explain their plan to each other and solve the problem together. Have them explain how their plans compared, and which was the best method for solving the problem. Have students assist their partners in finding words when necessary, using synonyms, and speaking with greater detail.
Author Intent
Students are given a scale drawing of a floor plan and the scale. The need to determine whether an object will fit in the room given its actual size. Students can convert the object’s dimensions to the scale drawing, or they can convert the scale drawing to actual dimensions.

Questions for Understanding

Dana Says (Screen 1) Use the Dana Says button to explain to students that, in a blueprint, a break in the line representing the wall shows a doorway.

• Why is it common to carefully mark doorways in blueprints? [Sample answer: You need to make sure that you don’t put furniture in the way of a door or have two doors that swing into each other.]

Before solving the problem

• What information do you need to know to solve the problem, and how can you find that information? [Sample answer: I need to find the actual dimensions of the section below the bathroom. I can write and solve two proportions, one to find the length of that section and one to find the width.]

While solving the problem

• The width of the section is 5 ft. Does that mean the bed won’t fit? [No; you still need to find the other dimension. You need one dimension to be greater than 6 ft and the other to be greater than 3 ft.]

After solving the problem

• Both dimensions of the bed are longer than the width of the hallway or the door. How do you get the bed down the hallway and through the door to put it in the bedroom? [Sample answer: The bed has a third dimension, its height. If you turn the bed on its side, it will probably fit into the bedroom.]

Solution Notes
Some students may not write a proportion to solve. They may see that 1 inch = 10 feet and know that \( \frac{1}{2} \) inch = 5 feet and \( \frac{3}{4} \) inch = 7 \( \frac{1}{2} \) feet using number sense. It is important for students to recognize that even if they solve the problem using mental math, they are using scales. Encourage them to confirm their work in writing.

Error Prevention
Students who are less comfortable with decimals may benefit from rewriting the dimensions of the scale drawing as fractions and then as decimals before solving the problem.

Got It Notes
In this problem, students find an area instead of two dimensions. They also need to read carefully to realize that the problem asks for two coats of paint.

Use a blank Know-Need-Plan organizer to help students plot the steps they must take to solve the problem. They need to find the actual length of the living room wall, find the area of the wall, and then double it. Finally, they need to compare that number to the area that a gallon of paint covers.
PART 3 (8 MIN)

Objective: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships and use to solve problems.

Author Intent

Students must compare the exchange rates for two different banks to see which offers the better rate. For Bank A, students are given a table, and for Bank B, students are given a graph. They need to find and compare the constants of proportionality and realize that a greater constant is a better rate.

Questions for Understanding

Before solving the problem

- How can you compare these exchange rates? [Sample answer: I can find the constant of proportionality for each bank. The greater constant will be a greater rate, which is more yuan for each dollar.]

Dana Says (Screen 1) Use the Dana Says button to explain that exchange rates at different banks are often different. Even if the difference is small, travelers will still want the best exchange rate.

- Why would you want to know which bank has a better exchange rate? [Sample answer: If you have a certain amount of money, you want to get the most yuan for your money. A greater exchange rate will get you more yuan for the same amount in dollars.]

Solution Notes

Students should notice that the best way to compare the exchange rates is to extract the constant of proportionality from each model. Because students are given that both relationships are proportional, they only need to find one ratio for each bank.

Got It Notes

Again students find the constant of proportionality for each situation, this time from an equation and from a graph. Though students know that \( \frac{1}{10} > \frac{1}{12} \), they may need to plug values into each equation to figure out the solution.

CLOSE AND CHECK (8 MIN)

Focus Question Sample Answer

Sample: You can represent the relationship with tables, graphs, equations, scales, and equivalent ratios. When you know different ways to represent the relationship, you can use the one that is most helpful in solving the problem.

Focus Question Notes

Listen for students to describe pros and cons of the various representations. They may list problems from this topic and tell which representation they would use for each problem.
Essential Question Connection

Use the questions below to help students remember that there are many ways to distinguish proportional relationships from ones that are not proportional. To be able to communicate with others and to understand a classmate or team member’s approach, it is a good practice to become comfortable with a variety of representations.

- Use one real-life example from this lesson to describe what constant of proportionality means. [Sample answer: In Example 3, the constant of proportionality is the number of Chinese yuan equal to one U.S. dollar. It is a unit rate. If a constant of proportionality exists between two quantities, then the quantities have a proportional relationship. This means that you can multiply one quantity (the number of dollars) by the constant of proportionality and the product will always be the corresponding value of the other quantity (the equivalent value of yuan).]

- Suppose you are given a table of values. How could you determine if the table shows a proportional relationship without graphing the points? [Sample answer: You could find each \( \frac{y}{x} \) ratio in the table. If they are all the same, then the relationship is proportional.]

- Suppose you are given an equation. How could you determine if the equation shows a proportional relationship? [Sample answer: If the equation is of the form \( y = mx \), it shows a proportional relationship.]

- In math, sometimes the first way you try to solve a problem doesn't work. In these instances, you have to be able to change course and try something new. Suppose you are given an equation that represents a proportional relationship. How would you make a graph that represents the relationship? What would be some characteristics of the graph? [Sample answer: You could use the equation to generate a table of values and then plot the ordered pairs. The graph would show a straight line passing through the origin.]