Writing Two-Step Equations

LAUNCH (7 MIN) ____________________________________________

Before
• Why might a baker want to have a lower price for an extra bagel?

During
• How many operations will you need in your equation? Why?

After
• How would the equation change if the 13th bagel cost the same as the others?

PART 1 (8 MIN) ____________________________________________

While writing the equation
• How many operations will you use in the equation? Explain.

After writing the equation
• Is the length of the average macaw less than or greater than the length of the average toucan? How do you know?

PART 2 (10 MIN) ____________________________________________

During the Intro
• What quantities affect the amount of the monthly payments?

Before solving the Example
• Why would the grower want to know how many oranges to pack in a box?

While solving the Example
• What is an expression for the weight of \( n \) oranges? What is an expression for the weight of the box? How are these expressions related?

Sara Says (Screen 2) Use the Sara Says button to point out that the box has weight before any oranges are placed in it, and to help students break the problem down into smaller parts.

PART 3 (7 MIN) ____________________________________________

While solving the problem
• How do the situations let you know which operations to use?

After solving the problem
• What are some other key words and phrases that you could use for addition?

Sara Says (Screen 1) Use the Sara Says button to connect this example about equations to a previous example about expressions. Strategies should be similar: look for key words to help you understand the relationships between the numbers and the variable.

CLOSE AND CHECK (8 MIN) _______________________________________
• How is a situation that is modeled by a one-step equation different from a situation that is modeled by a two-step equation?
• Now that you can write two-step equations, what do you think you will learn next about equations?
LESSON OBJECTIVE
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about the quantities.

FOCUS QUESTION
What kinds of problems call for two operations?

MATH BACKGROUND
In Grade 6, students modeled real-world situations by writing algebraic expressions and one-step equations. In the previous lesson, students reviewed solving one-step equations that contain rational numbers.

This lesson introduces students to two-step equations. The focus of the lesson is on writing two-step equations that model real-life problems or represent verbal sentences. Students learn how to identify the key words and phrases in a problem that signal how to write the equation.

In this lesson students do not solve the two-step equations they write. The next lesson teaches students how to solve two-step equations by applying the properties of equality they learned earlier in the topic.

LAUNCH (7 MIN) ________________________________________________________________
Objective: Write an equation involving two operations to represent a real-world problem.

Author Intent
Students write a two-step equation using their own methods and reasoning, prior to learning this skill more formally in the Examples. It is important to see how students represent situations on their own before teaching them a method for doing so.

Questions for Understanding
Before
• Why might a baker want to have a lower price for an extra bagel? [Sample answer: It may encourage people to buy bagels from that bakery or to buy more bagels to get the extra one at a lower price.]

During
• How many operations will you need in your equation? Why? [Sample answer: You will need two operations. You have 12 bagels with the same price, so the total cost of these bagels can be represented using multiplication. The 13th bagel has a different price, so you must use addition to include its price in the total.]

After
• How would the equation change if the 13th bagel cost the same as the others? [Sample answer: The equation would be a one-step equation since it would involve only multiplication.]

Solution Notes
Although the equation in the provided solution involves multiplication and addition, some students may use only addition in their equation: 

\[ b + b + b + b + b + b + b + b + b + b + b + 0.13 = 5.53 \]

Acknowledge that this equation is correct, but also remind these students that multiplication is the same as repeated addition, and help them write the equation as 

\[ 12b + 0.13 = 5.53. \]
Some students may instinctively realize that removing the cost of the extra bagel, $.13, leaves the cost of the other 12. These students may write the equation as

$$12b = 5.53 - 0.13.$$ 

**Error Prevention**

Some students may mistakenly write the equation $13b = 5.53$, which assumes that each bagel costs the same amount. Have these students find the value of $b$ to see that each of the 13 bagels would cost about $.43. Explain that this can’t be correct because you know the extra bagel costs $.13.

**Connect Your Learning**

Move to the Connect Your Learning screen. Start a conversation about how you need more than one operation to solve some problems. Discuss the Launch, and how one knows it requires more than one operation. Students should begin to recognize that a problem with more than one action on the unknown requires more than one operation.

**PART 1 (8 MIN)**

Objective: Write equations of the form $px = r$ or $p + q = 4$, where $p$, $q$, and $r$ are specific rational numbers.

**Author Intent**

Students write a one-step equation to represent a real-world problem in preparation for writing a two-step equation in the next Example. The equation in this problem involves rational numbers.

**Instructional Design**

Have a volunteer come to the whiteboard and circle the important information in the problem. The Words to Equation organizer is embedded in this Example. Use this organizer to help students translate from words to math symbols.

**Questions for Understanding**

While writing the equation

- How many operations will you use in the equation? Explain. [Sample answer: The equation will have one operation. The toucan’s length is a fraction of the macaw’s length, which can be modeled by multiplication.]

After writing the equation

- Is the length of the average macaw less than or greater than the length of the average toucan? How do you know? [Sample answer: The length of the average macaw is greater than the length of the average toucan because the toucan’s length is only $\frac{2}{3}$ of the macaw’s length.]

**Solution Notes**

The key phrase in the problem statement is “two thirds the length of.” Emphasize that when one quantity is a fraction of another quantity, the relationship between the two quantities can be expressed using multiplication.

**Got It Notes**

If you show answer choices, consider the following possible student errors:

Students who choose A or C are using the wrong operation. Stress that the correct equation involves multiplication because the amount of meat each lion eats times the number of lions equals the total amount of meat.
Students who choose D are using the correct operation but have interchanged the total amount of meat and the number of lions.

**PART 2 (10 MIN)**

Objective: Write equations of the form \( px + q = r \), where \( p, q, \) and \( r \) are specific rational numbers for real-world situations.

### ELL Support

**Beginning**

Use the model in the Intro as pre-reading support to assist students as they write an equation to solve the Got It. On the board, write the word equation: weight = initial weight + weight gain per month \( \times \) number of months. Have students read this equation before they read the Got It problem. Students then find the correct numbers to replace the words. \([1,000 = 125 + 65x]\). Then help students solve the equation.

**Intermediate**

Have students do the same activity as the Beginning learners. Then change the numbers in the word problem. Have them write an equation and solve it.

**Advanced**

Have students write the word equation that they can use to solve the Got It. \([\text{weight} = \text{initial weight} + \text{weight gain per month} \times \text{number of months}]\) Then have them write the number equation to match the word equation. \([1,000 = 125 + 65x]\) Then solve the problem.

### Author Intent

The Intro defines two-step equation and shows students how to write a two-step equation for a problem situation using the Words to Equation organizer. In the Example, students apply what they learned in the Intro to write a two-step equation for a new situation. The Example extends the modeling students have done with one-step equations.

**Instructional Design**

Play the animated Intro to introduce students to two-step equations. Encourage students to explain the problem in the Intro in their own words so that they understand why multiplication and addition are used. If they think the word remainder in the problem is a key word that suggests division, substitute a phrase like “the rest of the money” to avoid confusion.

Move to the Example, which has the Words to Equation organizer embedded. Invite two students to come to the whiteboard. One student can underline the important information in the problem statement and put the words in the Words to Equation organizer. The other students can translate the words into an equation.

### Questions for Understanding

**During the Intro**

- What quantities affect the amount of the monthly payments? [the amount of the down payment and the number of monthly payments]

**Before solving the Example**

- Why would the grower want to know how many oranges to pack in a box? [Sample answer: With this information, the grower can figure out the total number of boxes needed to ship a given number of oranges.]
While solving the Example

- What is an expression for the weight of \( n \) oranges? What is an expression for the weight of the box? How are these expressions related? [0.2\( n \); 10; the sum of the expressions is 50.]

**Sara Says (Screen 2)** *Use the Sara Says button to point out that the box has weight before any oranges are placed in it, and to help students break the problem down into smaller parts.*

**Solution Notes**

The provided solution uses addition and multiplication to write the equation. However, some students may reason that they can subtract the weight of the box from the total weight to get the weight of the oranges, and write the equation \( 0.2n = 50 - 10 \). Let these students know that their reasoning is correct.

**Differentiated Instruction**

*For struggling students:* To help students understand how to incorporate the decimal in this equation, use what students know about ratio reasoning. Ask students how many oranges there are in 1 kg of oranges. Once they understand that there are 5 oranges per kg, they may better understand how to write the equation.

*For advanced students:* Suppose the grower begins buying boxes that use lighter materials and weigh only 7 kg. How many additional oranges can be packed in such a box? Assume that the total weight of the box and oranges must still be 50 kg.

**Got It Notes**

The provided solution gives the equation \( 1000 = 125 + 65n \) as the answer. Students should realize that this equation is equivalent to the correct answer \( 1000 = 65x + 125 \) in choice B. Explain that any letter (not just \( x \)) can be used to represent the number of months, and that the terms 65\( x \) and 125 can be switched in the equation due to the Commutative Property of Addition.

If you show answer choices, consider the following possible student errors:

- Students may choose A if they think the calf’s initial weight (rather than the weight gained per month) should be multiplied by the number of months.
- Students may choose C if they think the calf’s initial weight should be subtracted from (rather than added to) the weight the calf gains.

**PART 3 (7 MIN)**

**Objective:** Choose the appropriate equation that represents quantities in a real-world or mathematical problem.

**ELL Support**

On the Student Companion page for the Part 3 Got It, there are three tasks for students to complete and discuss:

- Divide the word sentence of the problem into three parts.
- Read each part out loud.
- Then show how you wrote each word phrase using math symbols.

**Beginning**

Work through the first two tasks as a group, using highlighting and underlining to differentiate the parts of the word sentence. Then have students complete the third task.
Intermediate
Have students complete the first task independently. Then have them work in pairs to review their responses and to read through the parts of the word sentence together. Then have them complete the third task.

Advanced
Have students complete the three tasks. Then have them write a real-world problem situation for the equation they wrote in the Part 3 Got It.

Author Intent
Being able to translate words into mathematical symbols is one of the keys to modeling with mathematics. The given problem helps teach this skill by having students match equations to verbal sentences. The equations and sentences all use the same numbers, so students must interpret the key phrases in the sentences to decide which operations should be used in the equations.

Instructional Design
Call four students to the whiteboard and have each student drag an equation to the correct situation. Ask each student to explain why they think their equation belongs with a given situation. The rest of the class can indicate whether they agree with the four students’ choices before the answers are submitted.

Questions for Understanding
While solving the problem
• How do the situations let you know which operations to use? [Sample answer: The situations contain key words and phrases such as more than, product, and difference that indicate which operations to use.]

After solving the problem
• What are some other key words and phrases that you could use for addition? [Sample answers: sum, plus, greater than, total.]

Sara Says (Screen 1) Use the Sara Says button to connect this example about equations to a previous example about expressions. Strategies should be similar: look for key words to help you understand the relationships between the numbers and the variable.

Solution Notes
The provided solution uses the Words to Equation organizer to help students translate the sentences into equations. Point out that the word is in each sentence represents the equal sign, and the number following is appears on one side of the equal sign (while the other quantities appear on the other side).

Error Prevention
Students sometimes assume that the order of the numbers or expressions in a sentence must match the order in the corresponding equation. This is not always the case. In part (d), for example, the word seventeen appears before twice a number in the sentence, but 17 appears after 2x in the corresponding equation.

Got It Notes
If you show answer choices, consider the following possible student error:

Students who choose A may be assuming that the order of 8 and 10n in the equation must match the order of eight and ten times a number in the sentence. Explain that eight less than ten times a number means 10n - 8, not 8 - 10n.
CLOSE AND CHECK  (8 MIN)

Focus Question Sample Answer
Problems in which there are two different things acting on the unknown quantity are often modeled with a two-step equation. Two-step equations usually have two operations.

Focus Question Notes
Look for students to identify that one of the operations is usually addition or subtraction, and the other is multiplication or division. Key words in the problem help you determine which operations to use.

Writing an equation with two operations is similar to writing an expression with two operations, but now you know what the expression is equivalent to, so you can write an equation.

Essential Question Connection
This lesson addresses the Essential Question about when it is useful to model a relationship with an equation.

- How is a situation that is modeled by a one-step equation different from a situation that is modeled by a two-step equation? [Sample answer: A situation that is modeled by a one-step equation has only one thing acting on the unknown. It uses one operation. A situation that is modeled by a two-step equation has two things acting on the unknown, so it requires two operations.]
- Now that you can write two-step equations, what do you think you will learn next about equations? [Sample answer: How to solve two-step equations.]